Christ Church Grammar School

2021
TEST 3

MATHEMATICS METHODS Year 12

Section One:
Calculator-free

Your name _____

Teacher's name_____

Time and marks available for this section

Reading time before commencing work:	2 minutes
Working time for this section:	15 minutes
Marks available:	15 marks

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

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Instructions to candidates

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2. Write your answers in this Question/Answer Booklet using a blue / black pen. Do not use erasable or gel pens.

3. Answer all questions.

4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

7. It is recommended that **you do not use pencil**, except in diagrams.

See next page

Question 1 **(6 marks)**

Evaluate the following.

(a) $\int_1^2 \frac{d}{dx}(x^2e^{2x})\,dx$ (2 marks)

(b) $\frac{d}{dx}(\sin(2x) + e^{4x^2})$ (2 marks)

(c) $\int -2xe^{2x^2}dx$ (2 marks)

See next page

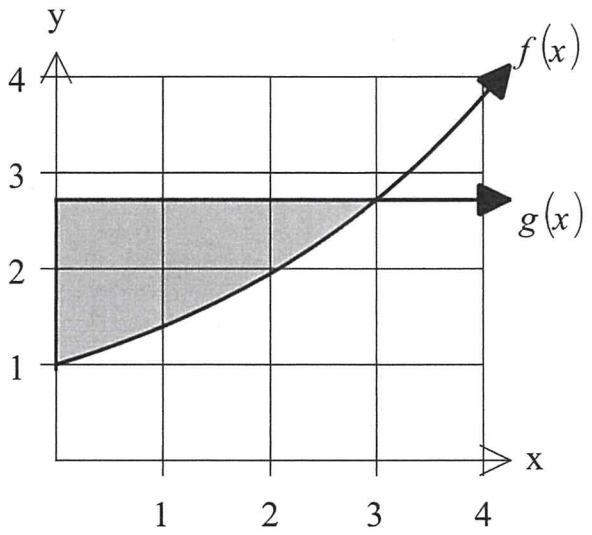
Question 2 **(3 marks)**

Given $\frac{d}{dx}(xe^x - e^x) = xe^x$, determine exactly $\int_0^1 (xe^x + x^2)dx$.

See next page

Question 3 **(3 marks)**

The functions $f(x) = e^{\frac{x}{3}}$ and $g(x) = e$ are graphed below, intersecting at $(3, e)$.



Determine the area bound by the two curves and the y-axis.

Question 4 **(3 marks)**

Determine the x coordinates of all stationary points of the function:

$$f(x) = \int_0^{4x} e^{t^2} - e\, dt$$

End of questions

Additional working space

Question number: _____

Christ Church Grammar School

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TEST 3

MATHEMATICS METHODS Year 12

Section Two:
Calculator-assumed

Your name _____

Teacher's name _____

Time and marks available for this section

Reading time before commencing work:	3 minutes
Working time for this section:	30 minutes
Marks available:	30 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

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See next page

Question 5 **(6 marks)**

Given that $X \sim B\ [n, p]$, (i.e. a Binomial Distribution) with $Var(X) = 1.5$ and $E(X) = 2$.

(a) Determine the value of n and p. (4 marks)

(b) Determine an expression for $P(X \geq 1)$. Do not simplify your answer. (2 marks)

See next page

Question 6 **(5 marks)**

A population grows continuously such that $\frac{dP}{dt} = 0.09P$, where P is the size of the population t years after observation commenced. When observation commenced, the size of the population was $350\ 000$.

(a) Determine an expression for P in terms of t. (1 mark)

(b) How long will it take for the population to reach $1\ 000\ 000$? (2 marks)

(c) A second population is increasing at the same rate as that from part (a), however the initial population is unknown. How long will it take for this population to double in size? (2 marks)

See next page

Question 7 **(4 marks)**

The concentration, C mg/KL, of a chemical in the CCGS pool, at time t weeks is given by $C = 0.2(1 + 8t)e^{-t}$, for $0 \leq t \leq 8$. Determine

(a) the exact value of t when the instantaneous rate of change of C with respect to t is 0. (2 marks)

(b) the exact maximum concentration of the chemical and state when this occurs. (2 marks)

See next page

Question 8 **(9 marks)**

Consider the discrete probability distribution shown below.

x	-1	0	1	2	3
$P(X = x)$	0.25	0.3	0.1	p	q

(a) Determine

(i) $P(X > -1)$ (1 mark)

(ii) $P(X = -1 \mid X \leq 1)$ (2 marks)

(b) Given that $E(X) = 0.85$, determine

(i) the values of p and q. (3 marks)

(ii) $E(2X - 1)$ (1 mark)

See next page

Question 8 continued

(c) Given that $SD(X) = 1.6$, determine

(i) $SD(3X)$ (1 mark)

(ii) $Var(3X)$ (1 mark)

See next page

Question 9 **(6 marks)**

It is known that 5% of cars manufactured in the Bitsaremissin assembly line will have some kind of defect.

(a) If a random sample of 8 cars is selected for testing, find that probability that

(i) no cars have a defect. (1 mark)

(ii) exactly 3 cars have a defect. (1 mark)

(iii) no cars have a defect, given that less than 5 cars have a defect. (2 marks)

(b) What is the largest number of cars that can be selected in a random sample such that the probability of there being at least 1 defective car is less than 20%? (2 marks)

End of questions

Additional working space

Question number: _____

Additional working space

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MATHEMATICS METHODS Year 12

Section One:
Calculator-free

Your name - SOLUTIONS -

Teacher's name _____

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See next page

Question 1 **(6 marks)**

Evaluate the following.

(a) $\int_1^2 \frac{d}{dx} x^2 e^{2x}\, dx$ (2 marks)

$= \left[x^2 e^{2x}\right]_1^2$ ✓ [Recognises FTC]

$= \underline{4e^4 - e^2}$ ✓ [ANSW]

OR $e^2(4e^2 - 1)$

OR $e^2(2e-1)(2e+1)$

(b) $\frac{d}{dx}(\sin(2x) + e^{4x^2})$ (2 marks)

$= \underline{2\cos(2x) + 8x e^{4x^2}}$

✓ ✓ [Each part]

(c) $\int -2xe^{2x^2} dx$ (2 marks)

$= -\frac{1}{2}\int 4x e^{2x^2} dx$

$= \underline{-\frac{1}{2} e^{2x^2} + c}$ ✓ (ANSW)

✓ (+C)

6

Question 2

Given $\frac{d}{dx}(xe^x - e^x) = xe^x$, determine exactly $\int_0^1 (xe^x + x^2)dx$. (3 marks)

$\therefore \int xe^x\,dx = xe^x - e^x + c$

So $\int_0^1 xe^x\,dx + \int_0^1 x^2\,dx$ ✓ [Splits Integral]

$= \left[xe^x - e^x\right]_0^1 + \left[\frac{x^3}{3}\right]_0^1$ ✓ [Uses given info]

$= e^1 - e^1 - [0 - e^0] + \frac{1}{3} - 0$

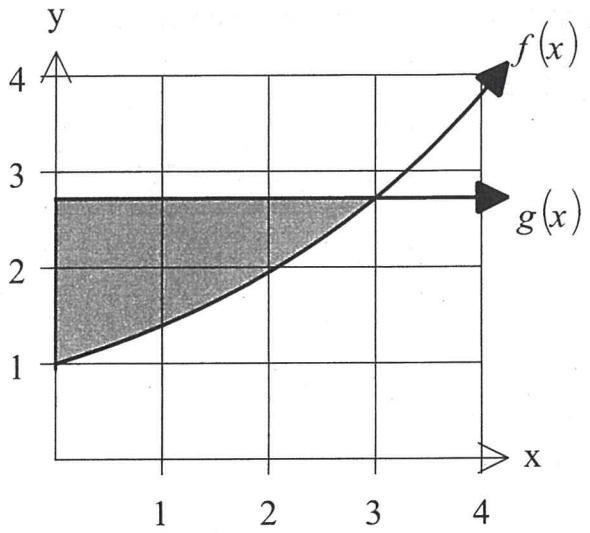
$= 1 + \frac{1}{3}$

$= 1\frac{1}{3}$ ✓ (ANSW)

3

Question 3 **(3 marks)**

The functions $f(x) = e^{\frac{x}{3}}$ and $g(x) = e$ are graphed below, intersecting at $(3, e)$.



Determine the area bound by the two curves and the y-axis.

$\int_0^3 e - e^{x/3}\, dx$ ✓ [correct integral]

$= \left[ex - 3e^{x/3}\right]_0^3$

$= 3e - 3e^1 - (e \times 0 - 3e^0)$ ✓ [sub values]

$= 3e - 3e + 3e^0$

$= 3 \text{ units}^2$ ✓ [ANSW]

3

Question 4 **(3 marks)**

Determine the x coordinates of all stationary points of the function:

$$f(x) = \int_0^{4x} e^{t^2} - e\, dt$$

$f'(x) = \left[e^{(4x)^2} - e\right][4]$ ✓ [FTC]

$f'(x) = 4\left(e^{16x^2} - e\right)$

Stat pt $f'(x) = 0$

$4\left(e^{16x^2} - e\right) = 0$ ✓ [$f'(x) = 0$]

$e^{16x^2} - e^1 = 0$

$e^{16x^2} = e^1$

$\therefore 16x^2 = 1$

$x^2 = \frac{1}{16}$

$x = \pm\frac{1}{4}$ ✓ [Both ANSW]

3

End of questions

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MATHEMATICS METHODS Year 12

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Calculator-assumed

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See next page

Question 5 **(6 marks)**

$X \sim B\,[n, p]$, with $Var(X) = 1.5$ and $E(X) = 2$. (i.e. A BINOMIAL DISTRIBUTION)

(a) Determine the value of *n* and *p*. (4 marks)

[Var equ] ✓ [E(X) equ] ✓

VAR(X) : $np(1-p) = 1.5$, $E(X)$: $np = 2$

$2(1-p) = 1.5$

$p = 0.25$ ✓ $n = 8$ ✓

(b) Determine an expression for $P(X \geq 1)$. Do not simplify. (2 marks)

$= 1 - 0.75^8$ ✓✓ (0.899887)

OR $\sum_{x=1}^{8} {}^8C_x (0.25)^x (0.75)^{8-x}$

6

Question 6 **(5 marks)**

A population grows continuously such that $\frac{dP}{dt} = 0.09P$, where P is the size of the population t years after observation commenced. When observation commenced, the size of the population was $350\,000$.

(a) Determine an expression for P in terms of t. (1 mark)

$P_t = 350\,000\, e^{0.09t}$ ✓

(b) How long will it take for the population to reach $1\,000\,000$? (2 marks)

$1\,000\,000 = 350\,000\, e^{0.09t}$ ✓ [Equ]

$t = 11.66469$ yrs

$\therefore\ t > 11.665$ ✓ [Must be $1\,000\,000^+$]

(c) A second population is increasing at the same rate as that from part (a), however the initial population is unknown. How long will it take for this population to double in size. (2 marks)

$\frac{2P_0 = P_0 e^{0.09t}}{2 = e^{0.09t}}$ ✓ [Equ]

$t = 7.70163$ yrs

$\therefore\ t > 7.702$ yrs. ✓ [Answ]

5

See next page

Question 7 **(4 marks)**

The concentration, C mg/KL, of a chemical in the CCGS pool, at time t weeks is given by $C = 0.2(1 + 8t)e^{-t}$, for $0 \leq t \leq 8$. Find:

(a) the exact value of t when the instantaneous rate of change of C with respect to t is 0. (2 marks)

$$\frac{-(8t-7)e^{-t}}{5} = 0 \quad \checkmark \text{ [Eqn]}$$

$t = \frac{7}{8}$ weeks ✓ [Answ]

(b) the exact maximum concentration of the chemical and state when this occurs. (2 marks)

EXACT MAX $C = \frac{8}{5}e^{-7/8}$ mg/KL (RAD) ✓ [MAX]

0.666979

When $t = \frac{7}{8}$ weeks ✓ [t-value]

[-1 overall if incorrect units]

4

Question 8 **(9 marks)**

Consider the discrete probability distribution shown below.

x	-1	0	1	2	3
$P(X = x)$	0.25	0.3	0.1	p	q

(a) Determine:

i) $P(X > -1)$ (1 mark)

0.75 ✓

ii) $P(X = -1 \mid X \leq 1)$ (2 marks)

$\frac{0.25}{0.65} = 0.3846$ ✓ [DENOM] ✓ [ANSW]

(b) Given that $E(X) = 0.85$ determine:

i) the values of p and q. (3 marks)

$\sum P(X=x) = 1 \quad \therefore p + q = 0.35$ -(1) ✓ [Equ (1)]

$-0.25 + 0 + 0.1 + 2p + 3q = 0.85$

$2p + 3q = 1$ -(2) ✓ [Equ (2)]

Solve on CPad.

$p = 0.05$, $q = 0.3$ ✓ [MUST HAVE BOTH]

ii) $E(2X - 1)$ (1 mark)

$= 2(0.85) - 1$

$= 0.7$ ✓ [ANSW]

7

See next page

Question 8 continued

(c) Given that $SD(X) = 1.6$, determine

(i) $SD(3X)$ (1 mark)

$3 \times 1.6 = 4.8$ ✓

(ii) $Var(3X)$ (1 mark)

$4.8^2 = 23.04$ ✓

2

Question 9 **(6 marks)**

It is known that 5% of cars manufactured in the Bitsaremissin assembly line will have some kind of defect.

(a) If a random sample of 8 cars is selected for testing, find that probability that

(i) no cars have a defect. (1 mark)

$0.95^8 = 0.6634$ ✓

(ii) exactly 3 cars have a defect. (1 mark)

CPAD 0.0054165 ✓

(iii) no cars have a defect, given that less than 5 cars have a defect. (2 marks)

$\frac{0.6634}{0.9999846}$ ✓ $= 0.6634$ ✓

OR $\frac{0.6634}{1}$

(b) What is the largest number of cars that can be selected in a random sample such that the probability of there being at least 1 defective car is less than 20%? (2 marks)

None defective > 0.8

$0.95^n > 0.8$ [equ] ✓

$n < 4.350345$

$n = 4$ ✓ [ANSW]

End of questions

Additional working space

Question number: _____

See next page

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See next page